International Journal of Foundations of Computer Science

TIGHT ANALYSIS OF SHORTEST PATH CONVERGECAST IN WIRELESS SENSOR NETWORKS *

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We consider the convergecast problem in wireless sensor networks where each sensor has a reading that must reach a designated sink. Since a sensor reading can usually be encoded in a few bytes, more than one reading can readily fit into a standard transmission packet. We assume that each packet hop consumes one unit of energy. Our objective is to minimize the total energy consumed to send all readings to the sink. We show that this problem is NP-hard even when all readings are of fixed size. We then study a class SPEP of distributed algorithms that is completely defined by two properties. Firstly, the packets hop along some shortest path to the sink. Secondly, the nodes use an elementary packing algorithm to pack readings into packets.

Our main technical contribution is a lower bound. We show that no algorithm for UCCP that either follows the shortest path or packs in an elementary manner is a $(2-\epsilon)$-approximation, for any fixed $\epsilon > 0$. To complement this, we show that SPEP algorithms are $(2-\frac{3}{2k})$-approximation for UCCP and 3-approximation for CCP, where $k \geq 2$ is the number of readings that can fit within a packet. We conclude with some special cases and experimental observations.

*This work was partially done when the first and fifth author were affiliated with Tata Research Development and Design Centre, Pune, India. The experiments were conducted when the first author was visiting the Institute of Mathematical Sciences, Chennai, India. The second author is supported in part by the National Science Foundation under grant CNS-0720875. The fifth author is on lien to Chennai Mathematical Institute, Chennai, India. Preliminary version of this work appeared in the proceedings of CATS 2011.

1. Introduction

The convergecast problem has obtained prominence among sensor networks researchers because it fits well with the goal of sensor networks, which is to monitor and collect data about an environment. The focus has been to either minimize the time, the energy, or the dual-criteria of both time and energy required to complete the convergecast [4, 8, 10, 12, 13, 15, 16, 19, 18, 20]. Researchers have also exploited spatial locality in many real-life convergecast scenarios by aggregating the data and transmitting the representative values for sub-regions within the region being sensed [9, 14, 6, 11].

Convergecast typically works as follows in sensor networks. The sensor nodes need to send sensed data to a centralized sink via multiple hops. A sensor reading can usually be encoded in a few bytes, so more than one reading can fit into a standard transmission packet, but there is a limit on the total number of bytes that each packet can carry. Each reading has to stay intact along the way. This is different from sensor data aggregation where a function is performed over several sensor readings to, typically, generate one single representative value for each region being sensed [6]. While data aggregation is agreeable in many situations, under certain scenarios, applications would rather desire the collected data to be exact. This requirement is common in scientific data gathering as indicated in [3, 17]. We have a cost associated with each hop, which is independent of the number of readings in it. This is an acceptable assumption commonly used in the sensor network community, although more realistic radio model indicates that packet size does matter [7]. Consequently, we ask the question: can we pack the readings in common routes to minimize the number of hops?

More formally, we are given a connected graph $G = (V \cup \{\mathbf{sink}\}, E)$ that is both undirected and unweighted. An edge $e = (u, v) \in E$ implies that u can communicate with v and vice versa. Each vertex v has a single reading of integral number of bytes $s(v)$ that has to be reported to the appropriately denoted vertex **sink**. These readings must travel to the **sink** in packets that have a capacity of k bytes. To ensure simplicity in the routing protocol, we require each reading to be treated in an atomic manner, i.e., the bits of the reading should neither be modified nor be partitioned and transmitted in separate packets. Since the readings have to fit in the packets, we assume that $\forall v \in V$, $s(v) \leq k$. A packet consumes 1 unit of energy every time it hops from a vertex to a neighbor regardless of the total size of the readings in it. Our objective is to minimize the total energy consumed to send all the readings to the **sink**. We primarily seek distributed routing algorithms in which the individual nodes are unaware of the entire graph; they are only aware of their immediate neighbors. We call this the `Convergecast Problem` or the `CCP`. Since convergecast is often repeated several times (like hourly temperature collection), our focus is on minimizing the total energy in this repeated operation. Therefore, we allow a one-time preprocessing phase. This can be used for constructing distributed data structures like shortest path trees.

Notice that CCP combines aspects of both bin-packing and routing. In Theorem 1, we show that it is NP-hard even when the underlying graphs are restricted to a line or a tree of depth greater than 1. These are fairly simple reductions from set partition. So, we focus much of our attention on a simplification in which the size of each reading is exactly 1 byte. We call this the Unit Convergecast Problem or the UCCP. In practice, many wireless sensor applications such as room temperature monitoring for energy conservation only need to deploy simple sensors with one single sensing attribute. These sensors then report small constant-sized readings as directed. In our formulation, we normalize it to one byte. More importantly, UCCP helps us gain insight into the problem when the effect of bin-packing is minimal because up to k single-byte-sized readings can be trivially placed into a packet. Interestingly, we show that even UCCP is NP-hard.

We study a class of algorithms for UCCP called Shortest Path Elementary Packing (SPEP) algorithms. The members of this class are all valid algorithms for UCCP that have the following properties.

Shortest Path Property: An algorithm for CCP or UCCP is said to follow the shortest path property if every packet hop always moves the packet along some shortest path to the **sink**. We refer to algorithms that have this property as shortest path algorithms. Because we are concerned with the convergecast problem, this property, when present, will make the solution more intuitive. This is essentially geographic routing with greedy forwarding often used in wireless sensor networks [1]. Note also that even distributed networks, with a little preprocessing, can easily establish a shortest path tree as long as the graph is connected.

Elementary Packing Property: An algorithm for UCCP is said to have the elementary packing property if each vertex communicates at most one partial packet and all the other packets, if any, are full. Such algorithms are called elementary algorithms. An elementary algorithm ensures that each node repackages the readings in the most straightforward manner. It also ensures that communication overhead in the entire network is minimized. This is because minimal number of packets will be used, leading to minimal total number of bytes in all the packets is minimal, since each packet has a constant-size packet header. We extend the definition of elementary packing to CCP as well, but defer the precise definition to the appropriate section.

In Section 2, we establish some preliminary hardness results. We first show that that CCP is NP-hard even when the underlying graph is very simple. We then show that UCCP is also NP-hard. In Section 3.1, we focus on upper bounds on the approximation ratio that SPEP algorithms can achieve. We first prove that all algorithms in SPEP are $(2 - \frac{3}{2k})$-approximate algorithm for UCCP(for $k \geq 2$). Subsequently, we show that SPEP algorithms are 3-approximate for CCP. In Section 4, we prove a somewhat counterintuitive result. We use a non-trivial construction to show that any algorithm that *either* follows the shortest path property or the elementary packing property cannot guarantee a $(2 - \epsilon)$-approximation for UCCP. In Section 5, we explore the performance of SPT when the underlying graph is either a tree or a grid

and show that it is optimal in the former case and asymptotically optimal in the latter case. To complement our theoretical analysis, we analyzed SPT experimentally. We discuss our experimental results in Section 6. Finally, we provide some concluding remarks.

2. Preliminary Hardness Results

It is quite straightforward to see that both CCP and UCCP are NP-hard. CCP is NP-hard even for some of the simplest trees via a reduction from SET-PARTITION to CCP. This result is formalized in Theorem 1. Although the proof is rather straightforward, we include it for the sake of completeness.

Theorem 1. *CCP is NP-hard even if the underlying graph G is a straight line or a tree of depth at least 2.*

Proof. Recall that in SET-PARTITION, we are given a set $U = \{x_1, x_2, \cdots, x_n\}$ of integers. The question we ask is whether U can be partitioned into two subsets such that the sums of the elements in either subsets are equal. SET-PARTITION is known to be NP-complete [5].

We can reduce an instance of SET-PARTITION to CCP in two very simple ways as shown in Figure 1, which illustrates the case when the instance of SET-PARTITION has 8 elements. We assume, without loss of generality, that the elements of SET-PARTITION are integral values between 1 and k and add up to $2k$. To reduce from SET-PARTITION to CCP, we form an instance of CCP in which each element of U forms a reading in CCP and is assigned to a node in CCP.

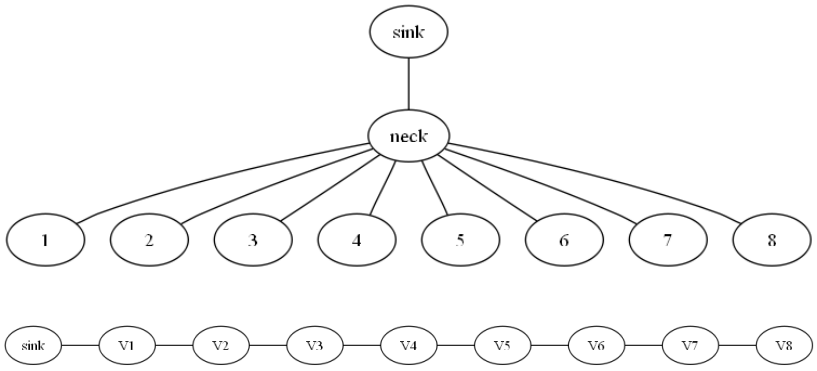

Fig. 1. The two figures illustrate the reductions from SET-PARTITION to CCP.

In the case of the tree of depth 2, we include a "neck" vertex which is assigned a reading of size k. The other nodes with readings assigned to them from SET-PARTITION are of degree 1 and are connected to the neck. The neck is connected

to the **sink**. The number of hops from the *neck* into the **sink** vertex will depend on whether the SET-PARTITION instance can be partitioned into two subsets.

Similarly, in the case of the line, the nodes form a linear chain with one end connected to the **sink**. Starting from the node farthest away from the **sink**, the readings travel toward the **sink**. At some point, there will be enough readings to require exactly 2 packets for any reasonable algorithm. Note that the **sink** has exactly one neighbour. Once all the readings reach that neighbour, we will need either 2 or 3 packets to hop into the **sink** depending on whether we can partition the set U or not. □

We now turn our attention to `UCCP`. We show that even `UCCP` is NP-hard by reducing the set cover problem to it. In the classic Set Cover Problem, we are given a ground set $U = \{x_1, x_2, \ldots, x_n\}$ and a family of subsets $S = \{S_1, S_2, \ldots, S_m\}$, $S_i \subseteq U$ for $i = 1, 2, \ldots, m$. $C \subseteq S$ is a cover if the union of elements in C is U. In the decision version of the problem, we are given a positive integer $K_{sc} < |S|$ and asked whether there is a subset of S with cardinality K_{sc} that covers U. It is well-known that Set Cover Problem is NP-complete [5].

Given an instance of the set cover problem, we construct a sensor network T consisting of vertices arranged in three levels as follows (refer Figure 2). Level 1 consists of only the **sink** node. Level 2 nodes correspond to the sets $S_i \in S$ for $i = 1, 2, \ldots, m$. There is an edge from each S_i to **sink**. We slightly abuse notation and use S_i to also refer to the corresponding vertex. Level 3 consists of nodes that correspond to $\{x_1, x_2, \ldots, x_n\}$ which are the elements of set U. Like level 2 nodes, we use x_j to refer to a level 3 vertex. Each node x_j is connected by an edge to S_i iff the element $x_j \in S_i$ in the Set Cover instance.

We set the size of a packet to $k = \max_i |S_i|$ bytes. We also add another $k-1$ leaf nodes, which we call enforcers, to each S_i. In Figure 2, the enforcers are depicted by a triangular pictorial gadget. Our objective is to solve the convergecast problem for this setup of sensor networks. i,e. each non-sink node (including the nodes in levels 2 and 3 and all the enforcers) has a reading of 1 byte and we must pass each reading to the **sink** using minimum number of packet hops.

For $K > 0$, we can show that $n + mk + K$ hops suffice to route each reading to the **sink** iff there exists a set cover of size less than or equal to K in the set cover problem. Each level 3 vertex has to send a packet to **sink** through a level 2 vertex. Note that at least n packets must hop out of the level 3 vertices for any solution (optimal or suboptimal). Consider the portion of the graph consisting of a single level 2 node S_i, its $k-1$ enforcers and **sink**. Regardless of the activity outside this portion, any solution requires k hops because the $k-1$ enforcers must communicate to S_i and we need a packet from S_i to the **sink**. Since there are m such level 2 vertices, the number of hops is at least mk. If at least one reading from level 3 vertex will hop through S_i, it will force S_i to send one more packet, which we call a *critical hop*. If $K \leq K_{sc}$ is the number of critical hops, then we can cover the ground set by selecting the subsets corresponding to each of the K chosen subsets.

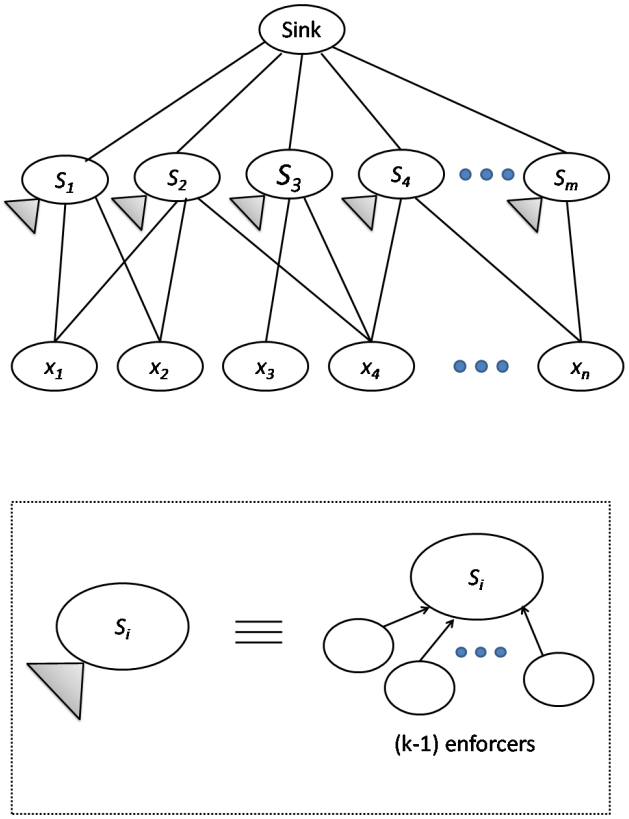

Fig. 2. Reducing the Set Cover problem to the UCCP. The enforcers are depicted as a triangular pictorial gadget; the actual construction of the enforcers is shown in the box.

Therefore, the following theorem follows.

Theorem 2. *UCCP is NP-hard.*

3. Upper Bounds

We now turn our attention to SPEP algorithms for UCCP and CCP and show that their approximation ratios are bounded by small constants

3.1. *The Shortest Path Elementary Packing Algorithms For UCCP*

While our focus is the entire class of algorithms (SPEP) for solving UCCP, we pick a canonical example from SPEP, the shortest path tree algorithm, or SPT, to prove

our results. Corollary 6 extends the results to all algorithms in SPEP.

The steps in the SPT algorithm are as follows. In the preprocessing phase, we construct a shortest path tree T of graph G rooted at **sink**[a]. As a consequence, each node is aware of its parent and children. Subsequently, each vertex waits till it has received all packets from its children in T. Full packets are sent to its parent as is. All the partial packets are re-packaged into the maximum number of full packets and at most one partial packet and all these packets are sent to the parent.

Let **OPT** and $\mathcal{A}$ be the number of hops taken by the optimum solution and SPT, respectively, in solving an instance of the UCCP. We show that $\mathcal{A} \leq (2 - \frac{3}{2k})\mathbf{OPT}$, where $k \geq 2$.

The maximum number of readings that can be packed in a packet is k. If a packet contains k readings then we call it a *full packet*; otherwise, it is a *partial packet*. If a full packet hops from a node a to neighbouring node then we will term this as *full hop*. A partial hop is defined likewise. We split **OPT** into $\mathbf{OPT}^f$ and $\mathbf{OPT}^p$ such that they are the number of full and partial hops, respectively. We define $\mathcal{A}^f$ and $\mathcal{A}^p$ in like manner. Naturally,

$$\mathbf{OPT} = \mathbf{OPT}^f + \mathbf{OPT}^p \tag{1}$$

$$\mathcal{A} = \mathcal{A}^f + \mathcal{A}^p. \tag{2}$$

Let us define the depth $d(v)$ of node v as the shortest distance of node v from **sink** in T, i.e., the minimum number of hops required for a reading to reach **sink** from v. The following lemma holds for any algorithm that has the elementary packing property.

Lemma 3. *For any instance of the UCCP, $\mathcal{A}^p \leq 2 \cdot \mathbf{OPT}^p$.*

Proof. Consider the packets that flow through a single vertex v according to any algorithm regardless of optimality. There is at least one partial hop either out of v or into v. We can prove this by contradiction. Suppose there were no partial hops into v, but ℓ full hops into v. Then, $k \cdot \ell + 1$ readings would have to hop out of v, which requires at least one partial hop. This implies that at least $n/2$ hops are partial even for an optimal algorithm. Therefore,

$$\mathbf{OPT}^p \geq n/2. \tag{3}$$

According to the SPT algorithm, each vertex waits for all its children to communicate their packets and reorganizes the readings such that at most one packet is not full. Therefore, $\mathcal{A}^p \leq n$, which, along with Equation 3, completes the proof. □

Before we proceed into proving our theorem, we point out an obvious property (formalized in Lemma 4) of any algorithm that obeys the shortest path property, the SPT being one such algorithm. The reading corresponding to vertex v travels a

[a]We do not delve into the details of this construction as it has been studied in various contexts in the past. For example, see the work by [2].

distance of exactly $d(v)$, which is the shortest distance to reach the **sink**. Therefore, the sum of all the distances traveled taken over all *readings* (not packets) by SPT is less than or equal to that of any other algorithm. That sum is at least $\mathcal{A}^p + k\mathcal{A}^f$ for SPT; we pessimistically account only one reading to have hopped in each partial packet. Similarly, the sum of the distance moved by *readings* according to an optimal algorithm is at most $(k-1)\mathbf{OPT}^p + k\mathbf{OPT}^f$; we liberally account for $k-1$ readings in each partial hop. Therefore, we can state the property as follows:

Lemma 4. *For any instance of the SPT on UCCP,* $\mathcal{A}^p + k\mathcal{A}^f \leq (k-1)\mathbf{OPT}^p + k\mathbf{OPT}^f$.

Theorem 5. *For any instance of UCCP,* $\mathcal{A} \leq (2 - \frac{3}{2k})\mathbf{OPT}$.

Proof. Using Equations 1 and 2, we rewrite the equation in Lemma 4 as

$$\begin{aligned} k\mathcal{A} &\leq (k-1)\mathbf{OPT} + \mathbf{OPT}^f + (k-1)\mathcal{A}^p \\ &\leq (k-1)\mathbf{OPT} + \mathbf{OPT}^f + \mathbf{OPT}^p + \\ &\qquad (k-1)\mathcal{A}^p - \mathcal{A}^p/2 \quad \text{(using Lemma 3)} \\ &= k \cdot \mathbf{OPT} + (k - 3/2)\mathcal{A}^p. \end{aligned}$$

Recall that $\mathcal{A}^p \leq n$. Hence, we can replace $\mathcal{A}^p$ with **OPT** because **OPT** is at least n; every vertex has to send out at least one packet. Further, dividing by k on both sides, we get $\mathcal{A} \leq (2 - \frac{3}{2k})\mathbf{OPT}$. □

Theorem 5 proves the upper-bound for SPT, but the underlying lemmas, Lemma 3 and Lemma 4, are true for all algorithms in SPEP. Lemma 3 holds for any algorithm that packs its readings in an elementary manner and Lemma 4 is true for any algorithm that respects the shortest path property. Therefore we can state:

Corollary 6. *The approximation ratio of any algorithm in* SPEP *for UCCP is at most* $(2 - \frac{3}{2k})$*, where* $k \geq 2$*, and SPT is optimum when* $k = 1$.

Note that in SPT, each node sends its packets to its parent in the shortest path tree. In practice, we might not want to burden one parent. This can be alleviated by choosing any node that is one step closer to the sink. Conceivably, a node can send its packets to, say, either a randomly chosen node that is closer to the sink or choose one in round robin fashion. Corollary 6 ensures that such variants will not incur a higher hop-count than SPT. This can be of use to systems designers who are interested in balancing the network overhead across the network without compromising the hop-count.

3.2. *The Shortest Path Elementary Packing Algorithms For CCP*

As in the case of UCCP, SPEP algorithms for CCP follow the shortest path and employ elementary packing. While the shortest path property extends naturally, we adapt the definition of elementary packing to fit CCP. We say that a packet is light if

the sum of sizes of readings in that packet is at most $\lfloor k/2 \rfloor$. Otherwise, we call that packet heavy. An algorithm for CCP is said to have the elementary packing property if each vertex communicates at most one light packet and all the other packets, if any, are heavy. Notice that any non-elementary packing can be easily converted to an elementary packing by iteratively combining two light packets until the packing becomes elementary. Notice also that well-known (and commonly used) bin packing algorithms like First-Fit, First-Fit-Decreasing, Best-Fit, Best-Fit-Decreasing, etc., are all automatically elementary. Such algorithms that employ elementary packing for CCP are called elementary algorithms. Of course, SPEP algorithms for CCP are those elementary algorithms in which the packets always advance (strictly) closer to the **sink**. As in the case of UCCP, the canonical SPEP algorithm is the shortest path tree algorithm (SPT) in which a shortest path tree is built during the preprocessing phase and the packets travel along this tree and the nodes repackage the readings in an elementary manner.

Theorem 7. *Any* SPEP *algorithm is a 3-approximation for* CCP*.*

Proof. Consider any SPEP algorithm. We first establish some notations. Let $\mathbf{OPT}$ and $\mathcal{A}$ denote the number of packet hops incurred by an optimal solution and an SPEP algorithm, respectively. Let $\mathbf{OPT}_i^h$ and $\mathbf{OPT}_i^\ell$ be the number of heavy and light packets, respectively, that hop from level i to level $i-1$ in the optimal solution. Let $\mathbf{OPT}_i = \mathbf{OPT}_i^h + \mathbf{OPT}_i^\ell$, $\mathbf{OPT}^h = \sum_i \mathbf{OPT}_i^h$, and $\mathbf{OPT}^\ell = \sum_i \mathbf{OPT}_i^\ell$. These notations are extended to subscripted and superscripted variants of $\mathcal{A}$.

Notice that $\mathbf{OPT} \geq |V|$. Combining this with the elementary packing property we get

$$\mathcal{A}^\ell \leq \mathbf{OPT}. \tag{4}$$

We now focus on the packets hopping from level i to $i-1$. Let b_i be the number of bytes that must hop from level i to level $i-1$. Put another way, b_i is the number of bytes originating from nodes at level i or farther. From the definition of heavy packets, we can conclude that $\mathcal{A}_i^h \leq \frac{2b_i}{k}$ while $\mathbf{OPT}_i \geq \frac{b_i}{k}$. Thus, $A_i^h \leq 2 \cdot \mathbf{OPT}_i$, which, on summing over all levels gives

$$\mathcal{A}^h \leq 2 \cdot \mathbf{OPT}. \tag{5}$$

Therefore, $\mathcal{A} = \mathcal{A}^h + \mathcal{A}^\ell \leq 3 \cdot \mathbf{OPT}$ (from Equations 4 and 5). □

4. Lower Bounds on Approximating UCCP using SPEP Algorithms

Given the upper-bound on the approximation ratio of SPEP in Corollary 6, a natural question we ask is whether the analysis can be tightened. We are, however, interested in algorithms that use shortest paths and/or employ elementary packing. In this subsection, we discuss the inapproximability of UCCP when either one of those two properties must be respected.

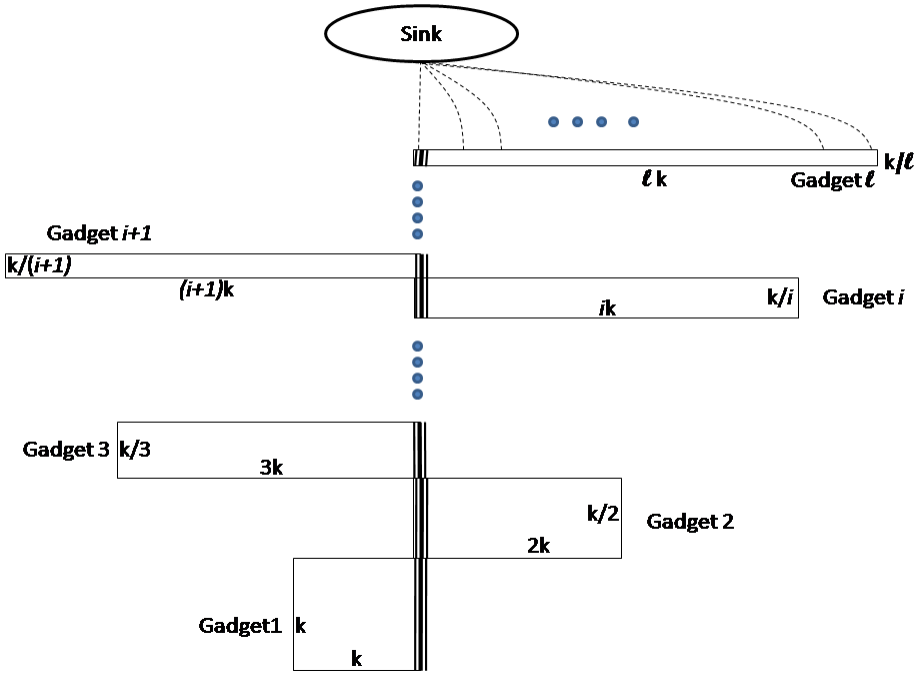

Fig. 3. The construction of an instance of UCCP used to prove Theorem 12 and Theorem 13. Note that the boxes are gadgets shown in Figure 4.

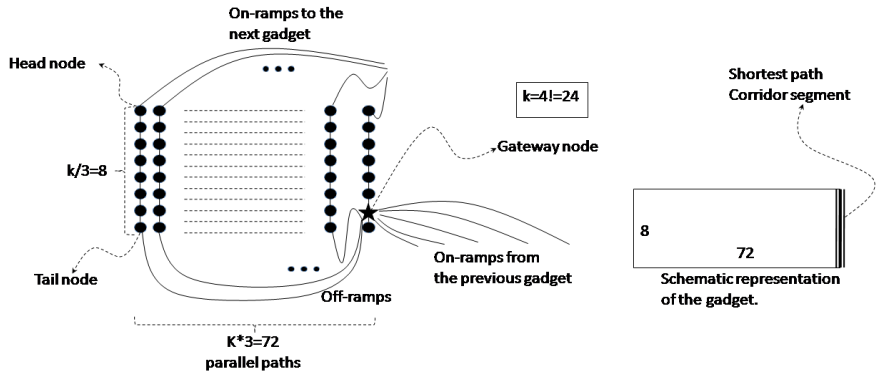

Fig. 4. A gadget for constructing the instance $\mathcal{E}_{\{\ell\}}$ of UCCP. The schematic representation of the actual gadget is also provided in the bottom right.

We begin by describing the construction of an instance $\mathcal{E}_{\{\ell\}}$ of the UCCP, where ℓ is a positive integer. This instance is constructed with one bad path (called the shortest path corridor or SPC) to the **sink** such that an optimal algorithm can avoid

it to minimize the number of hops. However, in the construction, we ensure that an algorithm that does not compromise on either the shortest path property or the elementary packing property cannot avoid the SPC and therefore must hop more.

The instance $\mathcal{E}_{\{\ell\}}$ will consist of ℓ gadgets (shown in Figure 3). The gadgets are indexed by i, $1 \leq i \leq \ell$. Gadget 1 is farthest away from the **sink** and gadget ℓ is closest to it. Figure 4 depicts the detailed construction of a single gadget. Two consecutive gadgets will be connected as shown in Figure 5. Note that gadget ℓ connects to the **sink** (see Figure 3). The position of the **sink** and the orientation of the instance depicted in Figure 3 indicates that the packets move "upward."

Given the value of ℓ, we define the size of each packet to be $k = \ell!$. We first describe a generic gadget that is used in constructing each of the ℓ gadgets. Figure 4 depicts the construction of a gadget i; the figure shows the actual construction and a schematic representation, which will be used subsequently. A gadget is defined by parameters i, its gadget index, and k, the capacity of the packets. It consists of ik parallel paths that are disconnected from each other (except for some special edges called off-ramps described later). Each of these ik paths consists of k/i nodes; note that k/i is an integer because $k = \ell!$ and $i \leq \ell$. Therefore, each gadget has k^2 nodes. The two end nodes in each of the paths is designated either as a head node or the tail node depending on whether it is closer to or farther away from the **sink**, respectively. Furthermore, one of the ik paths is a special path that is called a "segment of the shortest path corridor" and is shown by thick triple lines in the schematic. When the gadgets are put together to form the entire instance, these segments will join to form a sequence of segments from the farthest gadget (away from the **sink**) all the way to the **sink**. This sequence of segments form the shortest path corridor or SPC.

In each gadget, the node connected to the tail node of the segment of the SPC plays a special role; in Figures 4 and 5, they are depicted as star shaped nodes. We call them gateway nodes because all packets enter a gadget through its gateway node. Borrowing from the terminology used in highways in the United States, the $(i-1)k$ edges coming into the gateway node from gadget $i-1$ are called on-ramps. There are $ik-1$ edges going from the gateway node to the tails in the gadget (except for the tail of the segment of the SPC). These edges are called off-ramps. See Figure 5 for a depiction of two consecutive gadgets along with how they are connected; again, the schematic representation is also provided. To construct the entire instance, the gadgets are placed one on top of the other such that their individual segments of the shortest path corridor align and form the full shortest path corridor that extends from gadget 1 all the way to gadget ℓ and then connects to the **sink**. This construction of the entire instance is depicted in Figure 3.

Lemma 8. *There is a solution to the convergecast problem on the instance depicted in Figure 3 that hops at most $k^2\ell + k\ell^2$ times.*

Proof. The solution works as follows. Each gadget has k^2 nodes. Therefore, gadgets

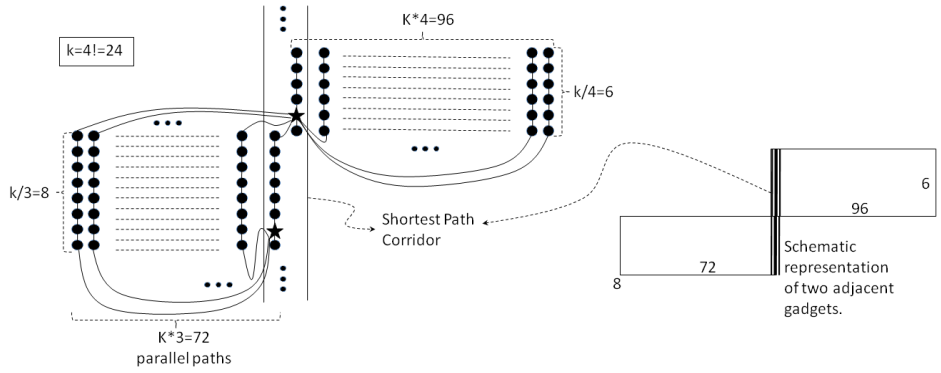

Fig. 5. Connecting two gadgets in adjacent gadgets. The box figure on the bottom right is the schematic representation for the actual construction in the top left.

1 to i have ik^2 readings that enter the gateway of gadget $i+1$. Then the gateway node, instead of sending them up the SPC, redistributes these packets to each of the $(i+1)k$ lanes in the gadget at level $i+1$. Therefore, each lane gets a packet that contains $\frac{i}{i+1}k$ readings that travel up each lane collecting the $k/(i+1)$ readings in that lane. Therefore, at the top of each lane in gadget $i+1$, the number of readings is $\frac{i}{i+1}k + \frac{k}{i+1} = k$, hence forming a full packet. These $(i+1)k$ full packets hop into the gateway at gadget $(i+2)$ and proceed toward the **sink** in like manner (i.e., avoiding the SPC and taking the lanes). Note that at gadget i, the following hop types occur. Firstly, the gateway node at gadget i feeds $(i-1)k$ packets (that it received from gadget $i-1$) via the off-ramps to the tail nodes in gadget i. This takes $ik-1$ hops; although there are ik paths, there is no need for a hop from the gateway to the segment of the SPC. Secondly, the ik packets travel up the lanes costing k/i hops per lane. This adds up to k^2 hops. Note that this includes the on-ramp hops that will carry the packets from gadget i into the gateway of gadget $i+1$. Therefore, at each level i, we incur a cost of $k^2 + ik - 1$. Considering this over all ℓ levels, the total cost is at most $k^2\ell + \sum_{i=1}^{\ell}(ik-1) \leq k^2\ell + k\ell^2$. □

Note that the cost incurred by the solution described in Lemma 8 hinges on the ability of the gateway nodes to pack in a non-elementary fashion. Hence it is not elementary in nature. Also, since it uses the off-ramps, it is not a shortest path solution either. We shift our concern to solutions that either use the shortest path or are elementary in nature. The key intuition here is that such solutions will transmit all the readings entering the gadget at level i only through the SPC. While the solution in Lemma 8 was able to split the $k(i-1)$ full packets into ik partial packets and ride up the gadget (in some sense, for free), the restricted solution will have to pay for these packet hops up the SPC. We dissect this cost in Lemma 10 and Lemma 11. Before that, we state Lemma 9, a simple observation about the instance $\mathcal{E}_{\{\ell\}}$.

Lemma 9. *The tail nodes (except those of the* `SPC` *segments) have exactly two shortest paths to the* **sink**. *All other nodes (including the tail nodes of* `SPC` *segments) have exactly one shortest path to the* **sink**.

Proof. The tail nodes that are not in the `SPC` segments can go through the gadget in two ways. They can either go via the off-ramps into the `SPC`, or go through the paths for which they are the tails. All other nodes, it is easy to see, have just one choice. □

The `SPT` incurs a higher hop count than the algorithm described in the proof of Lemma 8. Lemmas 10 and 11 formalize this limitation of `SPT`. The proofs of either lemmas show that their respective assumptions (namely, shortest path and elementary packing) force packets to take the `SPC`, which in turn forces them to hop at least $2k^2\ell - k^2 \log \ell$ times.

Lemma 10. *Any shortest path solution to the instance* $\mathcal{E}_{\{\ell\}}$ *depicted in Figure 3 requires at least* $2k^2\ell - k^2 \log \ell$ *hops. This holds regardless of whether the shortest path solution is deterministic or randomized.*

Proof. Each gadget produces k^2 readings because that many nodes are present in the gadget at that level. This has two consequences. Firstly, the number of hops within a gadget, not counting the hops of packets entering the gadget but counting the off-ramp hops, is at least k^2. The total number of such hops over all ℓ gadgets is $k^2\ell$. Secondly, the k^2 readings originating in gadget i must each travel a distance of $(k/(i+1) + k/(i+2) + \cdots + k/\ell)$, where each term accounts for the height of gadget $i+1$ up to gadget ℓ. We call these the `SPC` hops because these readings must travel up the `SPC`. Any alternate routing will violate the shortest path property. Hence, we can argue (in similar lines as in Theorem 5) that any optimal shortest path solution will form k full packets at the gateway node of gadget $i+1$. Hence, the total number of packet hops will be $k[(k/(i+1) + k/(i+2) + \cdots + k/\ell)]$. The total number of `SPC` hops originating over all ℓ gadgets is

$$\begin{aligned} k^2 \ [\ & (1/2 + 1/3 + 1/4 + \cdots + 1/\ell) + \\ & (1/3 + 1/4 + \cdots + 1/\ell) + \cdots + (1/\ell)] \\ = k^2[(\sum_{i=2}^{\ell} & \frac{i-1}{i})] \cong k^2[\ell - \log \ell]. \end{aligned}$$

Therefore, the total number of hops is at least $k^2\ell + k^2[\ell - \log \ell] = 2k^2\ell - k^2 \log \ell$.

We note here that a randomized shortest path solution does not have much flexibility because of Lemma 9. The readings from the tail nodes have two choices. However, any tail node that takes the off-ramp into the `SPC` will contribute to the two types of hops mentioned regardless of the choice it makes. If it goes through the `SPC`, it might contribute to more. Therefore, they are better off traveling through

their individual paths. Hence randomization does not help in decreasing the number of hops. □

Lemma 11. *Any elementary solution to the problem instance $\mathcal{E}_{\{\ell\}}$ requires at least $2k^2\ell - k^2 \log \ell$ hops.*

Proof. To prove this, all we need is to show that the "best" elementary solution will essentially route packets to the **sink** in the same manner as described in Lemma 10. In other words, we need to show that all packets entering a gadget through the gateway node must travel through the SPC to the **sink**. The instance $\mathcal{E}_{\{\ell\}}$ is constructed such that only the gateway nodes have degree greater than 2. Therefore, to ensure that an algorithm for $\mathcal{E}_{\{\ell\}}$ is elementary, we only need to ensure that gateway nodes observe the elementary packing property.

Consider the gateway node in gadget $i+1$. The readings routed through this gateway can be subdivided into those readings that *must* be routed through the gateway and those that have an alternate route. We first consider the readings that have an alternate route and show that, for the purpose of analysis, they can be assumed to take the alternate route rather than through the SPC. The readings that have an alternate route are the readings that originate from nodes in gadget $i+1$ itself, but not in the segment of the SPC in that gadget. Consider all the readings from non-SPC paths in gadget $i+1$. They form $(i+1)k-1$ paths and each path is of length $\frac{k}{i+1}$. If these readings moved in the tail-to-head direction along the path they were in (instead of using the SPC), they would require exactly $\frac{k}{i+1}((i+1)k-1)$ hops, which equals the number of non-SPC nodes in gadget $i+1$. This implies that exactly one hop must be accounted for each node's readings. Since each node requires at least one hop, routing these readings in any other way will not improve the hop count. Further, this tail-to-head routing does not violate the elementary packing principle. Hence, for any elementary solution, we can always construct another solution in which the readings from nodes not in the SPC do not use the segment of the SPC in their gadget.

The readings that must go through the gateway node are as follows.

(1) It will receive ik^2 readings from gadgets 1 through i.
(2) It has its own reading, and
(3) it also receives 1 reading from the tail node in the segment of the SPC in gadget $i+1$.

The elementary packing property therefore requires that exactly 1 partial packet (containing exactly 2 readings) will hop out of the gateway node. Quite obviously, all the full packets (in any reasonable elementary algorithm) will follow the SPC. The partial packet will also move up the SPC because if it were to take the off-ramp and go up the gadget through any other path, it will only incur extra hops.

Now that we have shown that the elementary packing property forces the routing to be similar to the one shown in Lemma 10, we can invoke the mathematical

machinery in that lemma to conclude the proof. □

Theorem 12. *For any fixed $\epsilon > 0$, there is no $(2-\epsilon)$-approximation algorithm for* UCCP *that follows the shortest path property. This holds even if randomization is permitted.*

Proof. Using the number of hops counted in Lemmas 8 and 10 in the asymptotic sense, the approximation ratio for any shortest path algorithm is at least

$$\begin{aligned}\lim_{\ell\to\infty}\frac{2k^2\ell - k^2\log\ell}{k^2\ell + k\ell^2} &= \lim_{\ell\to\infty}\frac{2k^2(\ell - \frac{\log\ell}{2})}{k^2(\ell + \frac{\ell^2}{k})}\\ &\approxeq \lim_{\ell\to\infty}\frac{2(\ell - \frac{\log\ell}{2})}{\ell}\\ &\qquad\qquad (\text{since } k \gg \ell)\\ &= \lim_{\ell\to\infty}(2 - \frac{\log\ell}{\ell})\\ &= 2.\end{aligned}$$

Since the limit reaches 2 from below, the theorem holds. □

The following theorem also follows similarly except that we must use Lemma 11 instead of Lemma 10.

Theorem 13. *For any fixed $\epsilon > 0$, there is no $(2-\epsilon)$-approximation algorithm for* UCCP *that respects the elementary packing property. This holds even if randomization is permitted.*

5. SPT on Tree and Grid Networks

We now turn our attention to the performance of SPT on special cases based on the graph G.

Theorem 14. *SPT is optimal for* UCCP *when the underlying graph G is a tree.*

Proof. Since G is a tree, all the readings from the descendants of any vertex v (including v's reading) will have to pass through v. Suppose there are R_v such readings. Then any algorithm will have to transmit at least $\lfloor \frac{R_v}{k} \rfloor + \lceil \frac{R_v \pmod k}{k} \rceil$, which is precisely the number of packet transmissions out of v in SPT. Therefore, SPT is optimal with respect to the number of packet hops. □

Suppose the graph G is a grid with m rows and n columns and the **sink** is the vertex at $(1,1)$, i.e. at row 1 and column 1. Since we are interested in the asymptotic behavior, we assume that m and n are $\omega(k)$. Furthermore, without loss of generality, we assume that m and n are multiples of k. We show that SPT-G, an

implementation of SPT with a specific underlying shortest path tree designed for grids, is asymptotically optimal. Whether all underlying shortest path trees lead to asymptotic optimality remains open.

The specific shortest path tree for SPT-G on an $(m \times n)$ grid is as follows: we designate each edge in G to be "vertical" (resp. "horizontal") if it connects vertices from the same column (resp. same row). All vertical edges are included in the SPT tree; horizontal edges are included iff they are from row 1. Intuitively, the packets move up the columns until they reach the first row. Once they reach the first row, they move towards the **sink** along the first row. Note that in keeping with our definition of SPT, once a packet becomes full, it does not split. We formalize the performance of SPT-G in Theorem 15.

Theorem 15. *SPT-G is asymptotically optimal for UCCP when the underlying graph G is an $m \times n$ grid, provided m and n are in $\omega(k)$.*

Proof. We begin by evaluating h_{LB}, the lower bound on number of hops required by any algorithm. Consider a horizontal cut in G between rows i and $i+1$. There are $(m-i)n$ readings below this cut. All these readings must pass through this cut. Assume that they pass through in full packets. Therefore at least $(m-i)n/k$ hops will pass through the cut. Considering all such horizontal cuts, the number of hops crossing these cuts must be at least $\sum_{i=1}^{m}(m-i)n/k = \frac{mn(m-1)}{2k}$. Similarly, we can also construct vertical cuts which induce at least $\frac{mn(n-1)}{2k}$ row-wise hops. Therefore, any algorithms will require at least h_{LB} hops given by

$$h_{LB} = \frac{mn(m-1)}{2k} + \frac{mn(n-1)}{2k} = \frac{mn(m+n-2)}{2k}. \tag{6}$$

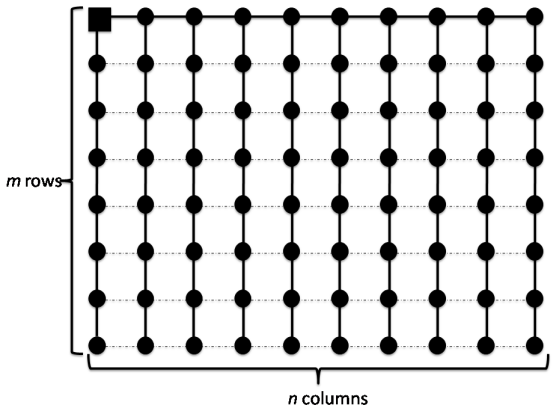

Fig. 6. SPT-G on grid. The square vertex is the **sink**. The full edges form the shortest path tree, while the dotted edges are discarded.

SPT-G starts with moving the packets up along columns. Once all the readings in a column are collected on the first row, the packets then move along the row to the **sink**. In each column, as the packets move upward, a new full packet is formed every k vertices. If we count all the partial hops in a single column, they are at most $m - 1 \leq m$. Since there are n columns, there are at most mn partial hops. Since the lower bound on the number of hops (from Equation 6) is $O(mn(m + n))$, the partial hops do not have any bearing on the asymptotic approximation ratio. Therefore, we are interested in evaluating $h^{\uparrow}$ and $h^{\leftarrow}$, which are the number of full packet hops up (along columns) and left (along rows) respectively.

There are at most $m/k - 1 \leq m/k$ full packets formed in each column. The first full packet is formed at row $m-k$ and full packets are formed regularly at an interval of k packets. From the vertex at which a full packet is formed, it will have to travel up to row 1. Therefore, the number of full packet hops in each column is at most

$$\begin{aligned}(m-k) + (m-2k) + \cdots + (m - (m/k)k & \\ \leq (m^2/k) - k(1 + 2 + \cdots + m/k) & \\ \leq (m^2/k) - k\frac{(m/k)^2}{2} & \\ = \frac{m^2}{2k}. &\end{aligned}$$

Since there are n columns in total, the number of hops up the columns, $h^{\uparrow}$, is at most

$$h^{\uparrow} = \frac{nm^2}{2k}. \tag{7}$$

Once the full packets reach the first row, they hop along the row towards the **sink**. Each column generates at most m/k full packets. Therefore, the total number of horizontal hops, $h^{\leftarrow}$ is given by:

$$\begin{aligned}h^{\leftarrow} &= (m/k)(1 + 2 + \cdots + n - 1) \\ &= (m/k)(n-1)(n)/2 \\ &\leq \frac{mn^2}{2k}.\end{aligned}$$

Therefore, the total number of full hops for $\mathtt{SPT} - \mathtt{G}$ is at most

$$h^{\uparrow} + h^{\leftarrow} = \frac{mn(m+n)}{2k}. \tag{8}$$

From Equations 6 and 8, it is clear that the upper bound and the lower bound converge asymptotically. □

6. Experimental Study

The lower bound for SPT is derived from pathological problem instances. It is quite likely that it actually does much better in practice. We would like to show this via

experimentation. For this purpose, we used JAVA to implement SPT. To objectively compare SPT, we computed three lower bounds on the number of hops for each input instance. They are:

LB1: The number of non-**sink** vertices $|V|$,
LB2: $\sum_{v \in V} d(v)/k$, where $d(v)$ is the shortest distance between v and the **sink**, and
LB3: $\sum_{i=1}^{D} n_i/k$, where $D = \max_{v \in V} d(v)$ and $n_i = |\{v : d(v) \geq i\}|$.

Each input instance was generated as follows. We defined an $N \times N$ euclidean region and placed nodes uniformly at random in this region. We normalized the radio frequency range to be 1. Therefore, a single hop link exists between two nodes that are within an euclidean distance 1 from each other. We deployed enough sensors such that the set of nodes and links formed a connected graph. We varied the value of N from 3 to 15 and for each value of N, we generated 50 input instances. We fixed $k = 5$. For each input instance I, we calculated the ratio, $r(I)$, of the number of hops in SPT over the largest of the three lower bounds. Figure 6 depicts the results of this experiment.

Based on the experimental results, we make a few observations. Firstly, the number of times the $r(I)$ values exceed 1.6 is very low. Secondly, such high values of $r(I)$ only occur in the relatively smaller instances. As the size of I is increased, $r(I)$ is lower with values around 1.3. Finally, We also note that the standard deviations of $r(I)$ for larger input instances are lower implying that the $r(I)$ values become stable as the input size increases. These observations clearly validate our claim that SPT requires few hops in more realistic input instances.

7. Conclusion and Future Work

Routing in sensor networks is quite complex owing to various constraints encountered in real instances. Our focus, therefore, was on a broad class of natural algorithms. This allows a network designer to construct specific algorithms within this class that fit her requirements. While algorithms in SPEP are at least 2-approximate (asymptotically), our experimental results indicate that they do quite well in practice. This leaves us with two questions for future work. Firstly, can we design algorithms with approximation ratios strictly less than 2 and 3, respectively, for UCCPand CCP? More importantly, will these improved algorithms (if designed) be intuitive, easy to implement and maintain, and worthy of competing in practice with SPEP algorithms?

Acknowledgement

We are thankful to M. V. Panduranga Rao and Dilys Thomas for participating in many discussions and providing valuable suggestions.

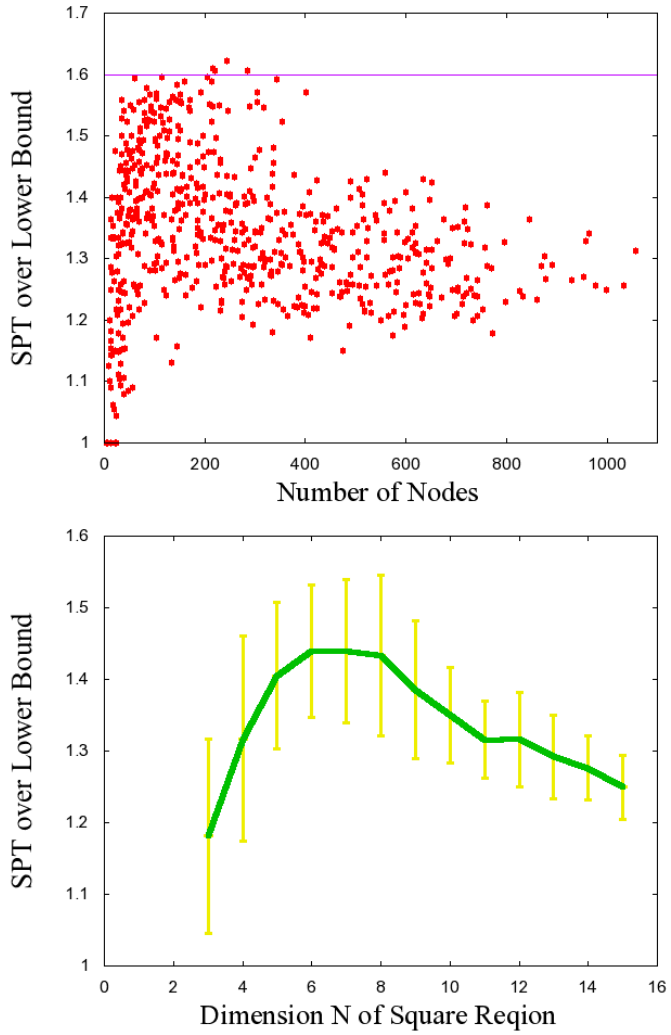

Fig. 7. The first graph is the plot of the $r(I)$ values against their corresponding number of nodes n. For the second graph, we generate 50 test instances for each value of N. The plot shows the average $r(I)$ values for each N. Additionally, we provide the standard deviation error bars.

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