On random routing in wireless sensor grids: A mathematical model for rendezvous probability and performance optimization

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1. Introduction

Wireless Sensor Networks (WSNs) are ad hoc networks of smart sensor devices for monitoring environmental or physical phenomena. These devices are expected to be wirelessly interconnected in large numbers to sense events, exchange data, and make collaborative decisions. Routing algorithms play a crucial role in self-organization and application dependent sensory data collection, but are constrained by energy and hardware (processing and memory) limitations.

Routing protocols for WSNs can be divided into two main categories: address-based and content-based. Address-based schemes rely on some type of network structure or node location information. A structure may be imposed via an initial self-organization phase, e.g., by forming a cluster tree [7] or by assigning logical coordinates [10,16]. Geographical location information may be obtained during deployment, or via a localization scheme [6]. Content-based routing protocols, in contrast, do not rely on geographical or logical coordinate systems; instead, a packet is flooded or forwarded to a randomly selected neighbor as in random routing [2,13,21,39]. Content-based routing uses content related attributes to define the destination set while address-based protocols rely on addresses for node identification and routing. For some applications, e.g., those with a central base station, the destination may be fixed or known by other nodes, while in other applications the sources and destinations may vary, or even unknown requiring their discovery. When source and destination are dynamic, content-based routing protocols such as rumor routing protocols [8,9,36,35], ant routing protocols [28,37], directed diffusion [18], and flooding [2,3] are used, whereas when the destination is known or fixed, address-based protocols are preferable. Certain applications use both content-based and address-based routing, e.g., content-based algorithms are used in an initial setup stage to discover the source of information and/or the destination, but then it switches to an address-based protocol for subsequent transfers. An example of this is a querying application in WSNs, where the initial destination discovery may be done using a random routing based search, with a different routing protocol used later in the query response stage. The content-based routing protocols play a vital role in WSN.
Simplicity, inherent load balancing, and the ability to trade-off energy vs. coverage are some of the properties that make random routing protocols attractive. As random routing does not guarantee the delivery or discovery of information of interest, one has to rely on models to tune the protocol parameters and to obtain the required level of reliability and power efficiency. Reliability here refers to the probability of a query meeting an event agent (agent). Rumor routing protocols [8,9,36,35], for example, use multiple agents and queries in order to achieve higher reliability. A formal model is necessary to select parameters such as the number of queries, number of agents, and Time-To-Live (TTL), i.e., the maximum number of hops a packet is allowed to traverse, to meet the performance goals such as WSN lifetime and application requirements under different scenarios. We present such a model for grid-based sensor networks.

Based on the research of [14,27,32], grid-based sensor networks are more suitable than other topologies for certain structural integrity monitoring, agriculture, environmental monitoring, surveillance, target location and tracking applications. Moreover [14,27,29,32] discuss the network capacity, robustness to failures, and optimal routing schemes of grid-based networks. Grid-based networks optimize the coverage, reliability, and sensing strength as well as energy consumption with the simplest architecture [24,38]. Because of the monitoring resolution in 3-D space [22,27,29], 3-D grids are preferred for spatiotemporal applications such as underground coal mine environments [27], forest monitoring [4], plume tracking [19] and underwater surveillance systems [11,30].

Modeling random routing in unstructured networks is difficult. Although uniformly distributed or Poisson distributed node placements may be used for modeling more complicated topologies representative of many practical networks, exact analytical modeling of random routing for such cases remains a challenge.

1.1. Our contributions

The model presented in this paper encompasses several scenarios. We start by deriving an exact closed form formula for the probability of a packet visiting a specific node after a given number of hops in random routing. Next, we extend the model to evaluate the probability of a query meeting an agent, resulting in a much simpler model than existing ones [12,25–27,34]. Use of the proposed model to evaluate the performance in the presence of multiple agents and multiple queries in a grid is illustrated. Basic relationships to extend the model to a 3-D grid is also derived. Physical boundaries of the network affect the random path taken by a packet; it is thus necessary to take into account the boundary conditions. We illustrate how to extend the analysis to the case with boundaries using a novel approach by considering a specific scenario. Finally, simulation and numerical results are used to demonstrate the validity and practical usefulness of the model.

The rest of our paper is organized as follows: Section 2 reviews some background and related work; Section 3 introduces the analytical model for rendezvous probability of an agent and a query; Section 4 presents simulation based verification of the model; Section 5 discusses the applications and the utility of the model; and Section 6 provides conclusions and future work.

2. Related work

Inherent properties of random routing, such as locality, simplicity, overhead, and robustness have attracted considerable interest. Random routing has been investigated from several different perspectives, e.g., for recently proposed novel routing protocols [1,5,8,9,23,31,33,36,35,39], or purely for mathematical modeling of random routing [12,25–27,34] for performance evaluation. Rumor Routing [9] protocol leads the path to many novel random routing protocols [8,36,35]. In Rumor Routing (RR), nodes that observe a physical event generate an agent carrying the information about the event. An agent is a packet with a fixed TTL (Time To Live) and carries information of one or more events. A query is a packet with fixed TTL that is generated by a node interested in the information of an event. RR protocol can be considered to lie in-between event (agent) flooding and query flooding, and it provides more flexibility for trading off energy vs. performance. An extension to RR, Zonal Rumor Routing (ZRR) [8], creates zones in the network so the agent/query can spread in the network more efficiently than with original RR, thus improving the query delivery rate and the energy efficiency. RR can be considered as a special case of ZRR where there is a single node in a zone. As another improvement to RR, [36,35] attempt to send the agent and queries in straight line paths using GPS and thus achieve a better delivery ratio with lower power consumption. The drawback of this method is the requirement of geographical information to send the packets in straight lines.

A routing method is presented in [1] for one-shot, pull-based queries, in which the sink issues queries for information in unstructured systems. This novel random routing algorithm has a three-way handshaking mechanism to improve reliability. Analysis in [1] is based on a simple random walk, where a packet is sent to all the neighbors, and self-avoiding random walk with k-memory. Moreover, they experimentally evaluate the performance in terms of reliability, delay and average energy on several sensor grids and have studied the performance in a realistic system. The evaluation in a real system is useful in assessing how much theoretical results deviate from the actual results due to real environmental issues that are difficult to model mathematically.

Use of random routing in query processing in dynamic environments is considered in [5], which presents a performance model in terms of energy consumption and the partial cover time, which is defined as time taken to cover a given fraction of the network. An upper bound on this partial cover time shows that for visiting a specified fraction of a sensor network, random walk is quite efficient in terms of the number of messages and the path length, and is also sufficient for answering many interesting queries with high quality. Random routing method proposed in [33] selects the next node according to a probability distribution that will result in a certain degree of load balancing in three different networks: a regular-static grid, an irregular-static grid, and a dynamic grid. ACQUIRE (Active Query forwarding In sensorN etworks) [31], a novel and efficient mechanism for gathering information in sensor networks, is most suitable for one-shot, complex queries for replicated data. A random walk based routing algorithm is proposed in [39], which also evaluates the probability of a packet moving between two nodes in a rectangular grid. This model however over-estimates the probability by neglecting certain path combinations and joint probability. The data gathering problem in a network with static nodes and a mobile patrol node is addressed in [23], which provides analytical bounds for coverage of an unconstrained random walk using a random geometric graph method on a square lattice.

Different approaches of modeling random routing protocols can be found in [12,25–27,34,39]. Random routing algorithms for n-D connected networks (i.e., networks with n inputs and n outputs per node) are analyzed in [12]. An iterative formula is derived to obtain the approximate throughput and link utilization of the random routing algorithm for 2-D connected networks using a Markov model, which is extended to obtain throughput and link utilization for n-D connected networks. The Brownian motion based model in [34] provides asymptotic results for three information querying strategies: source-only search, source-receiver “sticky” search, and spatially-periodic caching in a rectangular grid, whereas our...
model provides an exact solution for these cases using a much simpler model. The problem of random walk is addressed in [27] to model routing for data gathering in wireless sensor networks. An analytical model is proposed to evaluate the performance in terms of the mean system data gathering delay and the induced spatial distribution of energy consumption. It does not address the tuning of network parameters under design constraints or evaluate rendezvous probability of the agent and query. An analysis of the impact of the requisite knowledge on the routing efficiency in periodic lattice networks of finite square cells is provided in [25]. Finally, [26] quantifies the effectiveness of random routing in terms of the delay taken for the random walk taking place on a regular and periodic topology to deliver messages from sensors to sink nodes. Given the packet origin, the probability of a packet reaching a given node for the first time in the nth hop was derived using an asymptotic expansion of the probability of a packet being at a given node after given number of hops. Though this formula provides a good estimation of packet delivery delay, it is a complicated asymptotic evaluation and is too complex to be used for tuning network parameters.

Even though a significant amount of the literature addresses performance analysis of random routing using a specific metric, e.g., energy, none of them provides a general-purpose and flexible method of tuning the parameters to optimize performance in order to improve the WSN lifetime. Also asymptotic results provided for random routing in some prior studies are quite complex and are not applicable to static WSNs that we consider in this article. A brief comparison between our proposed model and existing models are given in Table 1.

### Table 1
Comparison of the proposed method and existing methods.

<table>
<thead>
<tr>
<th>Model</th>
<th>Applicable networks</th>
<th>Approximate/exact probability and parameters</th>
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<td>Proposed model</td>
<td>Small, medium, or large 2-D and 3-D loss/lossless grids without boundary and with regular boundary</td>
<td>Exact probability of agent meeting query, in terms of node position and TTL of the agent and query</td>
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<td>[25] Periodic lattice networks of finite square cells in 2-D</td>
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<td>[26] 2-D, periodic topology</td>
<td>Asymptotic results for source-only search, source-receiver sticky search and spatially-periodic caching</td>
<td>Probability of packet visiting to a given node (Over-estimates due to neglecting certain path combinations and joint probability)</td>
<td>Brownian motion</td>
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<tr>
<td>[27] Large scale 2-D WSNs with regular structure</td>
<td>Probability of none of the agents meeting any of the queries</td>
<td>Asymptotic expansion of the probability of a packet being at a given node after given number of hops</td>
<td>Brownian motion</td>
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<tr>
<td>[34] 2-D large rectangular grids</td>
<td>Probability of forwarding a packet to a neighbor in East, West, North, South</td>
<td>Asymptotic results for source-only search, source-receiver sticky search and spatially-periodic caching</td>
<td>Brownian motion</td>
</tr>
<tr>
<td>[39] 2-D rectangular grids</td>
<td>Probability of packet visiting to a given node (Over-estimates due to neglecting certain path combinations and joint probability)</td>
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<td>Brownian motion</td>
</tr>
</tbody>
</table>

3. **Analytical model for rendezvous probability of an agent and a query**

In this section, an analytical model for the probability of a query meeting an agent in random routing is derived. Section 3.1 provides a brief review of the routing protocol used. Consider a packet that is randomly routed on a rectangular grid. The probability of a query meeting an agent is obtained in three main steps:

1. **Derivation of** $P_H(I, J)$, **the probability of a packet visiting node** $(I, J)$ **in the** $H$th **hop.** After starting with a rectangular grid, we introduce results for a 3-D grid, and also provide means for accounting for packet loss probability and network boundaries. 

2. **Derivation of** $Q_H(I, J)$, **the probability of a packet visiting** $(I, J)$ **within** $H$ **hops, in terms of** $P_H(I, J)$. 

3. **Derivation of** $R_{H_q,	ext{tq}}|N_q,N_q'$, **the probability of a query meeting an agent when there are** $N_q$ **agents and** $N_q'$ **queries in the network, in terms of** $Q_H(I, J)$ **,** $H_q$ **is the agent TTL and** $h_q$ **is the number of hops traversed by the query, where** $h_q$ **is upper bounded by** $H_q$. 

Table 2 provides the definitions of major symbols. Next, the routing scheme used is briefly explained.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$(I, J)$</td>
<td>Physical coordinates of a node in 2-D grid</td>
</tr>
<tr>
<td>$(U, V, W)$</td>
<td>Physical coordinates of a node in 3-D grid</td>
</tr>
<tr>
<td>$H$</td>
<td>Time To Live (Maximum number of hops that a packet traverses)</td>
</tr>
<tr>
<td>$h$</td>
<td>Instantaneous hop count (≤$H$)</td>
</tr>
<tr>
<td>$e, w, n, s$</td>
<td>Number of movements in East, West, North, South</td>
</tr>
<tr>
<td>$P_{r,s}$</td>
<td>Probability of forwarding a packet to a neighbor in East, West, North, South</td>
</tr>
<tr>
<td>$u, q$</td>
<td>Subscripts denoting event agents and queries</td>
</tr>
<tr>
<td>$I_q, J_q$</td>
<td>Event location with respect to query origin</td>
</tr>
<tr>
<td>$H_q(H_q)$</td>
<td>Time To Live of agent (query)</td>
</tr>
<tr>
<td>$h_q(h_q)$</td>
<td>Instantaneous hop count 0 to $H_q$</td>
</tr>
<tr>
<td>$N_q$</td>
<td>Number of agents (queries)</td>
</tr>
<tr>
<td>$P_q(I, J)$</td>
<td>Probability of a packet visiting $(I, J)$ within $H$ hops</td>
</tr>
<tr>
<td>$Q_H(I, J)$</td>
<td>Probability of at least one of $N$ packets visiting $(I, J)$ within $h$ hops</td>
</tr>
<tr>
<td>$M_{H_q,N_q}(I, J)$</td>
<td>Probability of agent NOT meeting query within $H_q$ and $h_q$ hops at node $(I, J)$</td>
</tr>
<tr>
<td>$R_{H_q,N_q}$</td>
<td>Probability of agent meeting query anywhere for the first time within $H_q$ hops</td>
</tr>
<tr>
<td>$M_{H_q,N_q}(I, J)$</td>
<td>Probability of none of the $N_q$ agents meeting any of the $N_q$ queries within $H_q$ hops and $h_q$ hops</td>
</tr>
<tr>
<td>$R_{H_q,N_q}(I, J)$</td>
<td>Rendezvous probability of any of the $N_q$ agents and any of the $N_q$ queries anywhere for the first time at $h_q$th hop</td>
</tr>
<tr>
<td>$R_{H_q}$</td>
<td>Average reliability per node</td>
</tr>
<tr>
<td>$E_a$</td>
<td>Energy consumption per packet in a single hop</td>
</tr>
</tbody>
</table>

### 3.1 Random routing protocols

In random routing protocols, a node selects a random neighbor as the next hop for packet forwarding. Two types of randomly routed messages, namely, event agents and queries, exist in random routing algorithms such as rumor routing [8,9,36,35]. When a group of nodes experiences an event, each node probabilistically decides to send out a message, informing the rest of the nodes in the network. This message is called an “event agent” or simply an “agent”. On the other hand, a node looking for specific information...
of a physical phenomenon will send out a packet, called a query, requesting information. Both agents and queries are forwarded randomly until the TTL expires. Moreover each node that the agent visits keeps a record about the event for future retrieval by a query. Thus an agent and a query do not have to meet at a particular node at the same time. When the agent (query) meets query (agent) or a node which has the information, a path is discovered between source(s) and sink nodes. In order to increase the probability of the path discovery, the source (sink) nodes may send out more than one agent (query).

3.2. Probability of a packet visiting node \((I, J)\) in the \(H^\text{th}\) hop

3.2.1. Network without boundaries

Consider the rectangular grid shown in Fig. 1, where a node has four neighbors. We first analyze several scenarios without considering the network boundary, and then extend the analysis to consider a regular network boundary. Network boundaries can be ignored, for example, when TTL is small compared to the network size thus very few packets reach the network boundary.

A. Packet forwarded to a neighbor with equal probability, no packet loss

Without loss of generality, let the packet originating point be \((0, 0)\) and \(P_H(I, J)\) be the probability that the packet reaches node \((I, J)\), with \(I, J \geq 0\), in the \(H^\text{th}\) hop; \(H\) is the TTL of the packet. Due to symmetry, the probability of a packet reaching node \((I, J)\) is identical to that for \((-I, J), (I, -J)\) or \((-I, -J)\).

If \(H\) is less than \(I + J\) (\(=K\)), the packet cannot reach node \((I, J)\). When \(H \geq K\), the minimum number of hops required to reach \((I, J)\) is \(K\), and for the packet to end up at \((I, J)\), the remaining \((H - K)\) hops must be even. Let the total number of movements in east, west, north and south be \(e, w, n\) and \(s\) respectively, so that the total number of hops, \(H = e + w + n + s\). For the packet to end up at \((I, J)\), it should go net \(I\) hops in east direction, i.e., if \(w = i\), then \(e = l + i\). It should also go net \(J\) hops in north direction, thus if \(s = j\), then \(n = J + j\). The total number of movements in east, west, north and south directions are \(I + i, I - j, J + j, \) and \(J - j\) respectively, for non-negative integers \(i, j\).

Let \(P_H(I, J)\) be the probability of going to \((I, J)\) in the \(H^\text{th}\) hop. This follows the multinomial distribution [42],

\[
P_H(I, J) = \sum_{i+j=(H-K)/2} \frac{H!}{(I+i)!J!(J+i)!} \left( \frac{1}{4} \right)^H;
\]

\(i + j = (H - K)/2\). (1)

We use Vandermonde’s Convolution [20] to simplify this:

\[
P_H(I, J) = \frac{H!}{(M - I)!((M - J))!} \left( \frac{1}{4} \right)^H M = H + I + J.
\]

resulting in, \(P_H(I, J) = \left( \frac{H}{M - 1} \right) \left( \frac{H}{M} \right) \left( \frac{1}{4} \right)^H \). (3)

A similar approach yields the probabilities of a packet reaching \((U, V, W)\) in the \(H^\text{th}\) hop for 3-D case illustrated in Fig. 2.

There are six possible directions that a packet can move in a 3-D grid: East \((E)\), West \((W)\) North \((N)\), South \((S)\), Up, and Down. Let the destination be \((U, V, W)\). Similar to the 2-D case, the breakdown of the TTL can be written as

\[ U + 2u + V + 2v + W + 2w = H \]

where \(u, v, w\) are additional hops taken in \(E\) and \(W\), \(N\) and \(S\), Up and Down directions respectively. Let \(K = U + V + W\). Hence we can write \(u + v + w = (H - K)/2\) which is a constant.

\[ P[\text{Packet visiting } (U, V, W) \text{ at the end of } H \text{ hops}] = P_H(U, V, W) \]

\[ = \sum_{u=0}^{(H-K)/2} \sum_{v=0}^{H-K-u} \frac{H!}{(U+u)!(V+v)!(W+w)!u!v!w!} \rho^H. \]

\(\rho\) is 1/6 since there are six neighbors and selection of next node is equally probable.

\[
P_H(U, V, W) = \frac{H!}{M!} \rho^H \sum_{u+v+w=H-K} \left( \frac{1}{4} \right)^{u+v+w} \times \left( \frac{1}{4} \right)^{K+m} \frac{K+m}{U+u, V+v, W+w}. \]  

(4)

where \(u + v + w = (H-K)/2\), \(m\).

Eq. (4) can be used as the basis for the subsequent analysis of a 3-D network to derive the rendezvous probability by using an approach similar to the one given below for a 2-D grid. Thus we consider only the 2-D grid for the following analysis.

B. Packet forwarded to a neighbor (excluding the previous sender) with equal probability in a lossless network

This case is referred to as self-avoiding forwarding. Since the packet is not sent to the node that it was received from, the packet is routed to one of the remaining three neighbors on the rectangular grid with equal probability of 1/3. The derivation of a closed form exact result for \(P_H(I, J)\) in this case is difficult, while an iterative solution is possible. Therefore we first present an upper bound that can be used as an approximation or as a bound for subsequent analysis.

A path consists of an \(H\)-tuple, with each element in \((E, W, N, S)\). In this pattern, there are \(H - 1\) adjacent pairs. Let the number of
movements in N, S, E and W directions be \(n, s, e\) and \(w\); then the probability of occurrence of N, S, E, W in a hop are \(n/H, s/e, h\) and \(w/H\) respectively. If the packet could be forwarded with equal probability to any neighbor, we can enumerate the number of paths with \(n, s, e, w\) hops in the four directions and divide it by \(4^n\), the total number of random paths of length \(H\) to get the probability of ending up at \((i, j)\). However, as a packet is not forwarded to the node it came from, none of those adjacent pairs can be movements in \((N, S), (S, N), (E, W)\) or \((W, E)\). Thus there are a smaller number of legal paths. The probability of getting illegal consecutive hop pairs \((N, S)\) or \((S, N)\) as the first pair in the \(H\)-tuple is \(2(n/H(H-1))\), while the probability of getting \((E, W)\) or \((W, E)\) as the first pair within the \(H\)-tuple is \(2(e/H(H-1))\). This probability is the minimum possible probability of an illegal pair to occur within \(H\)-tuple.

As \(n = j + s = e = 1 + i\) and \(w = i\), the probability of a randomly generated path being valid for Case A (analyzed earlier, Eq. (3)), having illegal pairs here (Case B) is bounded below by,

$$2 \left( \frac{(i + j)i}{H(H - 1)} + \frac{(j + i)j}{H(H - 1)} \right).$$

Then we can write the probability of not getting an illegal pair for the first time within \(H\)-tuple as

$$1 - 2 \left( \frac{(i + j)i}{H(H - 1)} + \frac{(j + i)j}{H(H - 1)} \right).$$

This is the maximum possible probability that an illegal pair can occur anywhere within the \(H\)-tuple. Assuming the probability of getting an illegal pair anywhere in the \(H\)-tuple is independent and equally probable, an upper bound of the probability of any pattern not containing an illegal pair is given by

$$(1 - 2 \left( \frac{(i + j)i}{H(H - 1)} + \frac{(j + i)j}{H(H - 1)} \right))^{H-1}.$$ 

Now we can obtain the following bound for this case.

$$P_H(i, j) \leq \sum_{i=0}^{(H-1)/2} \frac{H!}{(i + j)i!(j + i)j!} \left( \frac{1}{3} \right)^{H-1} \times \left( 1 - 2 \left( \frac{(i + j)i}{H(H - 1)} + \frac{(j + i)j}{H(H - 1)} \right) \right)^{H-1}. \quad (5)$$

This is a tighter upper bound than that proposed in [17]. The effectiveness of this bound is evaluated in Section 4. Simplicity is the main advantage of having this upper bound compared to the exact probability evaluation. An iterative approach to evaluate the exact probability of this case (Eq. (5)) is explained next. 

C. Evaluation of exact probability of a packet visiting \((i, j)\) within \(H\) hops in self-avoiding forwarding

This section outlines a method to evaluate the exact \(P_H(i, j)\) for the case where next node is selected excluding the previous sender. Since a packet is not sent back to the previous sender, a hop in \(E\) cannot be preceded or followed by \(W\), and a hop in \(N\) cannot be preceded or followed by \(S\). This make certain hop patterns illegal. Let us define \(\overline{S}(h, i, j), \overline{N}(h, i, j), \overline{E}(h, i, j)\) and \(\overline{W}(h, i, j)\) as the number of valid paths that do not start with \(S, N, E\) and \(W\) respectively, \(h\) is the number of hops left, and \((i, j)\) is relative coordinate of the destination with respect to the current node.

It can be shown that

$$\overline{N}(h, i, j) = \overline{N}(h - 1, i, j + 1) + \overline{W}(h - 1, i, j - 1) + \overline{E}(h - 1, i + 1, j) \quad (6)$$

$$\overline{E}(h, i, j) = \overline{E}(h - 1, i + 1, j) + \overline{N}(h - 1, i, j + 1) \quad (7)$$

$$\overline{W}(h, i, j) = \overline{W}(h - 1, i - 1, j) + \overline{S}(h - 1, i, j - 1) + \overline{N}(h - 1, i, j + 1) \quad (8)$$

$$\overline{S}(h, i, j) = \overline{S}(h - 1, i, j - 1) + \overline{W}(h - 1, i, j - 1) + \overline{E}(h - 1, i + 1, j) \quad (9)$$

By considering the symmetry, Eqs. (6)-(9) can be replaced by

$$\overline{N}(h, i, j) = \overline{N}(h - 1, i, j + 1) + \overline{W}(h - 1, i, j) + \overline{E}(h - 1, i + 1, j). \quad (10)$$

Then the total number of legal paths, \(n_{H(i, j)}\) can be written in terms of remaining paths after first packet forwarding as,

$$n_{H(i, j)} = \overline{N}(h - 1, i, j - 1) + \overline{W}(h - 1, i, j + 1) + \overline{E}(h - 1, i + 1, j + 1) + \overline{S}(h - 1, i, j) \quad (11)$$

Eq. (11) is a iterative formula where each subcomponent of Eq. (11) can be rewritten using Eq. (10) where the boundary conditions are given as below,

$$\overline{N}(h, i, j) = \overline{N}(h - 1, i + 1, j) \quad (12)$$

After evaluating the total number of legal paths, \(P_H(i, j)\) can be evaluated as,

$$P_H(i, j) = n_{H(i, j)} \times \left( \frac{1}{3} \right)^{H-1} \quad (13)$$

D. Every node forwards the packet to one of its neighbors with equal probability with finite packet loss probability

The previous cases analyzed ignore the possibility of a packet getting lost. To consider how packet loss probability can be incorporated to the model, we next analyze a network where a packet is forwarded to one of the neighbors with equal probability (Case A), but with non-zero packet loss probability \(p_d\). Analogous to zero drop probability case, there should be net \(I\) number of hops to east, and \(J\) number of hops to north in order to end up at \((i, j)\) in \(H^{th}\) hop. But unlike in the case where the packet loss probability is zero, now a packet can reach \((i, j)\) even when \(H - K\) is odd due to packet drops. If a packet is lost, the TTL is reduced by one hop and the packet is retransmitted. Loss probability of a repeat transmission is the same as that of the initial transmission. Note that a node may verify that the packet was lost due to not hearing the forwarding of the packet by the next node. TTL is reduced to ensure that the power budget allocated is not exceeded. Let \(r\) be the number of hops in which a packet drop occurs. By following similar reasoning to Case A, that \(i\) and \(j\) hops taken in west and south directions requires \((i + j)\) hops in east and north directions respectively, the probability that packet reaches \((i, j)\) in \(H^{th}\) hop can be written as,

$$\hat{P}_H(i, j) = \sum_{r=0}^{H/2} \sum_{h=0}^{H/2} \frac{H!}{(i + j)!r!} \left( \frac{1}{4} (1 - p_d) \right)^{H-r} p_d^r; \quad (14)$$

where \(H - K - r\) is even.

To evaluate Eq. (14), we can use \(P_h(i, j)\)'s derived in Case A (Eqs. (3) and (4)) and Case B (Eq. (5)).
3.2.2. Network with boundaries

So far we have considered a network that is large compared to the TTL of the packets so that the packet will not reach a boundary. Next we present a novel approach that can take boundaries of regular shapes into account. Assume that a packet is forwarded to one of the neighbors with equal probability, and that there is no packet loss. When a packet reaches a boundary node, it sends the packet to one of its neighbors. This essentially is equivalent to packets being bounced back off the boundary into the interior. Without loss of generality, assume that the packet originates at (0,0). As shown in Fig. 3, packet origin can be anywhere within the rectangular region ABCD. Let ABCD be \(L_{x1}, L_{y1}, L_{x2}, L_{y2}\) hops away from the packet origin in east, west, north and south directions. If \(H\) is sufficient to exceed the boundary, when the packet is bounced by the boundary nodes, it will increase the probability of a packet ending up at \((I, J)\) in \(H^\text{th}\) hop. This increment in probability is the same as the probability of packet going to the mirror images of \((I, J)\) with respect to the boundary.

It is clear from Fig. 3 that the nodes in four quadrants are identical in the network with boundary ABCD. So we present the equations for the probability of a packet visiting the given node \(N_0 \equiv (I, J)\) with respect to packet origin in \(H\) hops: \(P_{H}^\text{boundary}(I, J)\), in the positive quadrant of the network.

Depending on the position of the node \(N_0\), that is whether \(N_0\) is on the boundary or on the edge or anywhere else, and the packet TTL \(H\), the possible number of mirror images that need to be taken into account changes. If \(N_0\) is at \(B\), then there can be three mirror images as \(I = I'\) and \(J = J'\). If \(N_0\) is on BC, there can be five possible mirror images since \(I = I'\) \((N_2, N_3, N_4, N_5, N_6)\) and \(J = J'\). Same is true when \(N_0\) is on DA. Similarly, if \(N_0\) is on AB, as \(J = J'\), there can be five possible mirror images since \(N_0, N_2, N_3, N_4, N_5, N_6\) overlap. Same is true when \(N_0\) is on CD. If the node is anywhere else there can be up to eight possible mirror images \(N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8\) that need to be taken into account. If \(H\) is not sufficient enough to reach any of the images, the corresponding probability term will disappear. On the other hand, if \(H\) is higher than \(\min(2L_{x1}, 2L_{y1}, 2L_{x2}, 2L_{y2})\), mirror images due to imaginary boundary \(\min(2L_{x1}, 2L_{y1}, 2L_{x2}, 2L_{y2})\) away from \((0,0)\) should be taken into account. Here we present the equations only for \(H \leq \min(2L_{x1}, 2L_{y1}, 2L_{x2}, 2L_{y2})\).

Therefore, the probability of the packet reaching \((I, J)\) in \(H^\text{th}\) hop when \(H \leq \min(2L_{x1}, 2L_{y1}, 2L_{x2}, 2L_{y2})\) is as follows:

\[
P_{H}^\text{boundary}(I, J) = \begin{cases} 
\frac{P_I(I, J) + P_{I'}(I', J') + P_{I''}(I'', J'') + P_{I'''}(I''', J''')}{4} & J = J' \neq J'' \neq J''' \neq J'''' \\
\frac{P_{II}(I, J) + P_{II'}(I', J') + P_{II''}(I'', J'') + P_{II'''}(I''', J''')}{4} & J = J' \neq J'' \neq J''' \neq J'''' \\
\frac{P_{II}(I, J) + P_{II'}(I', J') + P_{II''}(I'', J'') + P_{II'''}(I''', J''')}{4} & J = J' \neq J'' \neq J''' \\
\frac{P_{II}(I, J) + P_{II'}(I', J') + P_{II''}(I'', J'') + P_{II'''}(I''', J''')}{4} & J = J' \neq J'' \\
\frac{P_{II}(I, J) + P_{II'}(I', J') + P_{II''}(I'', J'') + P_{II'''}(I''', J''')}{4} & J = J' \\
\end{cases}
\]

where, \(I' = 2L_{x2} - I; J' = 2L_{y1} - J; I'' = 2L_{x1} + I; J'' = 2L_{y2} + J\) and for \((I + J), (I + J'), (I' + J), (I' + J'), (I'' + J), (I'' + J'), (I''' + J), (I''' + J')\) are not reachable by \(H\) hops, the corresponding probability that a mirror image in the above equations (Eq. (15)) disappears.

3.3. Probability of a packet visiting \((I, J)\) within \(H\) hops, \(Q_{H}(I, J)\)

A packet can end up at \((I, J)\) at the end of \(H\) hops by going through different paths. Such a packet may have visited \((I, J)\) in \(h = H - i\), where \(i \geq 0\) hops for the first time, and revisit it after the remaining \(i\) hops. Let \(p_{i}(I, J)\) be the probability of a packet visiting \((I, J)\) for the first time in the \(h^\text{th}\) hop. Then,

\[
p_{i}(I, J) = \sum_{h=0}^{H} P_{H-i}(I, J) P_{i}(0, 0)
\]

where, \(P_{i}(0, 0)\) is the probability of a packet revisiting itself within \(i\) hops, which can be obtained using Eq. (3) by substituting \(I = J = 0\) and \(H = i\) as

\[
P_{i}(0, 0) = \left(\frac{1}{1/2!}\right)^2 4^{i} 1^{i}.
\]

The probability of a packet reaching \((I, J)\) within \(H\) hops, is given by

\[
Q_{H}(I, J) = \sum_{h=0}^{H} p_{h}(I, J).
\]
In order to find \( Q_h(i, j) \) we need to find \( p_h(i, j) \) for all \( h \). Consider the matrix representation of \( P_h(i,j) \), where TTL is \( H \),

\[
[P_h(i,j)]_{1 \times 1} = [p_h(i,j) \ldots p_i(j) \ldots p_0(i,j)]_{1 \times H}
\]

For different hop values (from 0 to \( H \)), the matrix representation of \( P(I,J) \) can be written as,

\[
\begin{pmatrix}
P_0(I,J) \\
P_{H-2}(I,J) \\
\vdots \\
P_1(I,J) \\
P_H(I,J)
\end{pmatrix}_{(H-K)/2 \times 1}
= \begin{pmatrix}
P_0(0,0) & \ldots & P_{H-2}(0,0) & \ldots & P_{H-K}(0,0) \\
0 & \ldots & P_0(0,0) & \ldots & P_{H-K-2}(0,0) \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \ldots & 0 & P_0(0,0)
\end{pmatrix}_{(H-K)/2 \times 1}
\]

where \( K = I + J \). Thus,

\[
p_{k+2m}(i,j) = p_{k+2m}(i,j) + \sum_{i=0}^{m-1} p_0(0,0)p_{k+2i}(i,j);
\]

\[
i + j = m.
\]  

Eq. (18) is an iterative formula, with the initial condition given by Eq. (19),

\[
p_k(I,J) = p_0(0,0)p_k(I,J) = p_k(I,J).
\]  

In summary, first solve Eq. (18) iteratively for \( p_h(i,j) \) for \( h \) varying from \( K \) to \( H \), using the initial condition given by (Eq. (19)). Then \( Q_h(i,j) \) (Eq. (17)) can be evaluated.

### 3.4. Probability of a query meeting an agent

As the information exchange among nodes with random routing protocols such as rumor routing depends on queries meeting agents they seek, parameters have to be tuned to maximize this rendezvous probability. Recall that a node that detects an event sends one or more agents with a set TTL to inform other nodes about the event. Any node interested in that type of event will send one or more queries. Schemes such as rumor routing send more than one agent or query, thereby intending to improve performance. The rendezvous probability of an agent and a query anywhere in the network is derived below.

The probability of a query meeting an agent is zero when the sum of maximum hop counts of the two is smaller than the Manhattan distance between the event and query nodes. To apply the results derived above for probability of a packet reaching node \((I,J)\) with respect to its point of origination to both an event and a query, we define two frames of reference, one with respect to the node generating the agent, and the other with respect to the node generating the query.

Let subscript 'd' denote the agent and 'q' denote the query. Then, as stated in Table 2, \((I_d,J_d)\) is the agent location with respect to query origin, \(H_d(H_q)\) is the TTL of agent (query), and \(h_d(h_q)\) is the instantaneous hop count, which spans \([0 \text{ to } H_d(H_q)]\). Fig. 4 shows the coordinate system relationship with respect to the agent and the query, with the rendezvous point having coordinates \((l',j')\) and \((I,J)\) with respect to origin of query and agent respectively, where \(l' = l + 1\) and \(j' = j + 1\).

Without loss of generality, let the query origin be \((0,0)\) (see Fig. 4). By a query meeting an agent at a particular node we mean that both query and agent visit the particular node, but not necessarily at the same time. We define \( M_{h_d,h_q}(I,J) \) as the probability of the agent NOT meeting query at node \((I,J)\). In general, an agent generated travels until its TTL expires, e.g., see rumor routing protocol [9]. That is, agents are already spread in the network so that \(h_d = H_d\). Unlike agents, a query continues only till it meets an agent, i.e., it visits a node that an agent has visited, or its TTL expires, i.e., \(h_q \leq H_q\). Since spreading of agent and query are independent,

\[
P[\text{Agent NOT meeting query within in } h_q \text{ hops at } (I,J)] = 1 - Q_{h_q}(I,J).Q_{h_q}(l',j').
\]

Since the query not meeting an agent at node \((I,J)\) is independent of the query not meeting agent at \((I,J)\) within \(h_q\) hops, the probability of the agent not meeting query anywhere within \(h_q\) hops is given by,

\[
M_{h_d,h_q} = \prod_{(l,j) \leq h_q} [1 - Q_{h_q}(I,J).Q_{h_q}(l',j')].
\]  

Therefore, \( P[\text{a query meeting an agent anywhere for the first time within } h_q \text{ hops}] = R_{h_d,h_q} - R_{h_d,h_q} - 1 \).

Therefore, \( P[\text{a query meeting an agent anywhere} \) for the first time at \( h_{th} \text{ hop} = R_{h_d,h_q} - R_{h_d,h_q} - 1 \).

An advantage of Eqs. (20) and (21) is that we can evaluate the corresponding probabilities for different numbers of hops taken by agent, even prior to reaching its TTL. It is possible to design generalized versions of random routing protocols where the agent behavior is adaptive, and thus performance analysis would require this more general result.

Let the probability of at least one of \( N \) packets originating at \((0,0)\) visiting \((I,J)\) in \( h \) hops be \( Q_h^{(N)}(I,J) \). Since each packet is independent and identical, \( P[\text{any of } N \text{ packets visiting } (I,J) \text{ within } h-hops] \) is given by,

\[
Q_h^{(N)}(I,J) = 1 - (1 - Q_h(I,J))^N.
\]  

![Fig. 4. Coordinate system relationship for agent and query.](image-url)
After finding \( Q_h^{(N)}(I, J) \), using Eq. (22) for a given number of queries and agents, \( P[\text{Any of the } N_q \text{ agents and any of the } N_q \text{ queries NOT meeting any of the } N_q \text{ agents and any of the } N_q \text{ queries}] \) is given by

\[
M_{h_0, h_q} | N_q, N_q = \prod_{v(I, J) \leq N_q} (1 - Q_{h_0}^{(N_q)}(I, J) Q_{h_q}^{(N_q)}(I, J)).
\]  

(23)

So, the rendezvous probability of any of the \( N_q \) agents and any of the \( N_q \) queries anywhere in the network for the first time at \( h_0 \)th hop is given by

\[
R_{h_0, h_q} | N_q, N_q = 1 - M_{h_0, h_q} | N_q, N_q
\]

(24)

where, \( M_{h_0, h_q} | N_q, N_q \) can be evaluated using Eq. (23). Thus we have the probability of a query meeting an agent, in terms of the number of queries, number of agents, and the TTL values of queries and agents. Eq. (23) is valid for any small, medium, and large-scale grid, one of the main advantages of the model. In the next sections, we verify these results and illustrate the use of this model for tuning parameters in a network.

4. Simulation based verification

A Monte Carlo simulation of a rectangular grid of 2500 nodes, implemented in MATLAB 7.5, is used for the verification of the proposed model. The simulator itself was extensively verified using a number of standard techniques, including verifying traces and evaluation of known special cases. Unless it is explicitly stated otherwise, TTL \( H \) is set to 70. Without loss of generality, the packet generating node is assumed to be the origin. Due to symmetry, we present results only for nodes in the positive quadrant. Model verification was done for the following four cases, with no boundaries considered in the first three cases.

- **Case 1**: Exact \( Q_h(I, J) \) when next node selection is equally likely and there is no packet loss.
- **Case 2**: Upper bound for \( Q_h(I, J) \) when forwarding avoids the sender of the packet, and selection of node is equally likely among the remaining nodes. Packet losses are ignored.
- **Case 3**: Exact \( Q_h(I, J) \) when next node selection is equally likely with finite loss probability.
- **Case 4**: Exact \( Q_h(I, J) \) when next node selection is equally likely, in a network with zero packet loss, and a boundary.

4.1. Case 1: exact \( Q_h(I, J) \) when next node selection is equally likely and no packet loss

Fig. 5 shows the Monte Carlo verification for the probability of packet ending up at nodes on \( J = 0 \) cross section when \( H = 70 \). A similar agreement was observed for all other nodes as well. The probability is zero when the packet is unable to end up at \((I, J)\) at 70th hop. For \( H = 70 \) under lossless conditions, a packet cannot end up at a node when \( I + J \) is odd, and those zero values are not indicated in Fig. 5.

Recall that the formula of \( P_h(I, J) \) Eq. (3) is a product of two binomial distributions. Each binomial distribution can be approximated as normal (Gaussian) distribution [40]. Since approximated Gaussian variables \( I \) and \( J \) are independent, the product of Gaussian distributions will result in the form of jointly Gaussian distribution [41]. Therefore, \( P_h(I, J) \) given by Eq. (3) is proportional to joint Gaussian function of two independent random variables \( I, J \) with zero mean and equal variances of \( H/2 \) where the proportionate factor is 2. Thus the envelope of the discrete probability of packet ending up at \((I, J)\) node at the end of \( H \) hops follows a scaled version of jointly Gaussian distribution equation, and it is given by

\[
P_h^{\text{approximated}}(I, J) = \frac{2}{\pi H} e^{-\left(I^2+J^2\right)/2} \quad \text{for all } I, J \text{ and } H.
\]

(25)

**Fig. 6.** Variation of \( Q_h(I, J) \), the probability of a packet visiting \((I, J)\) within \( H \) hops, for a network with no packet loss: analytical values from Eq. (17) and Monte Carlo simulation based values.

**Fig. 5.** Monte Carlo based verification of probability of a packet reaching \((I, J)\) at \( H^{th} \) hop \( P_h(I, J) \) in Eq. (3) in \( J = 0 \) plane and its Gaussian equation approximation (Eq. (25)).
probability of a packet visiting \((I, J)\) within \(H\) hops obtained using Eqs. (3) and (17), corresponds to the 100\% node presence in Fig. 8. As can be seen, probabilities for 90\% and 80\% densities obtained via Monte Carlo simulation are almost the same as those for 100\% node availability case. Additional simulation results indicate that if the availability of nodes is more than 75\%, \(Q_H(I, J)\) is more or less the same as that for 100\% node available grid, thus making it possible to apply the model for such arrays as well. In sparse arrays some nodes may have all the four neighbors while other nodes have less. In fact, this indicates that the model presented may be used to evaluate a broader class of networks. For example, a network in which nodes go to sleep mode periodically is similar to a sparse-array.

4.3. Case 3: exact \(Q_H(I, J)\) when next node selection is equally likely, and with non-zero loss probability

To verify the model for the case with packet loss, a network with 20\% probability of packet loss at each node is simulated using Monte Carlo simulation, and the results compared with analytical results (Eqs. (14) and (17)) as shown in Fig. 9. Additional simulations, not presented here, confirm the validity of the model for the full range of values of \(p_d\).

4.4. Case 4: exact \(Q_H(I, J)\) when next node selection is equally likely and no packet loss

In a Monte Carlo simulation, a packet with TTL 100 is randomly sent in network with boundaries at \(|I| = 20\) and \(|J| = 20\) (\(L_x = L_y = 20\) in Eq. (15)). The probability of a packet visiting \((I, J)\) within 100 hops is evaluated and compared with the results of the analytical model, i.e., Eqs. (15) and (17), as shown in Fig. 10. As mentioned earlier, Eq. (15) holds only when \(H \leq \min (2L_x, 2L_y)\), because if \(H > \min (2L_x, 2L_y)\), there will be some more mirror images that are not captured by the Eq. (15). But as can be seen from Fig. 10, even though \(H (=100) > \min (2L_x, 2L_y) = 40\), Eq. (15) still holds, as the probabilities of ignored mirror images are very low, and in most cases they are almost zero. Hence ignoring those components did not make results of Eq. (15) deviate from exact probabilities given by Monte Carlo simulation (see Fig. 10). But if \(H\) is large enough so that probabilities at those ignored mirror image locations are significant, then there will be a noticeable error in \(Q_H(I, J)\) due to neglecting those components. However, a simple algorithm can be developed to include other mirror images in calculating this probability for such cases.

5. Applications of the analytical model

The main importance of the proposed model is its utility to set parameters of random routing protocols to achieve optimum performance under required conditions, e.g., for a given reliability, for fixed total energy, or for a given delay bound. In the following, we illustrate how to obtain the number of agents and number of queries that optimize the performance under a fixed energy budget. Two scenarios are considered: (A) a lossless network, with fixed packet length; and (B) a lossy network, with fixed packet length. The parameter settings for each of the cases are summarized in Table 3. A third case of a lossless network, where packet length varies as it propagates can be found in [17,15].

In evaluating the above three cases, the number of agents generated was varied from one to three, and for each case, the

---

Fig. 7. Variation of \(Q_H(I, J)\), the probability of a packet visiting \((I, J)\) within \(H\) hops, for a network with no packet loss: analytical values from Eq. (17) and Monte Carlo simulation based values.

Fig. 8. \(Q_H(I, J)\) on \(l = 0\) plane — analytical values for 100\% node presence and Monte Carlo simulation based values for 90\%, 80\% and 50\% node presence.

Fig. 9. Probability of a packet visiting \((I, J)\) within \(H\) hops, \(Q_H(I, J)\), when packet loss probability is 0.3.
The variation of Fig. 12 and Fig. 13 illustrates the reliability per node. The rendezvous probability, we use simulation to evaluate the performance of the agent visiting nodes in the area of interest is lower than that when $N_a$ agents are sent, each with TTL of $H_a/N_a$ thus utilizing the same power. Hence we can conclude that under fixed agent energy, the best performance is given by a single agent. In the simulation, TTL of a single agent $H_a$ is set to 170 and packet loss probability is set to zero.

For convenience, we use the convention $(N_a, N_q)$ to denote the cases where agent is spread initially, and $(N_a, N_q)$ for the case when queries are spread initially in the network. The other packet type (i.e., query and agent respectively) is spread after the network is stabilized. For example $(N_a, N_q) = (3, 1)$ indicates that three agents were initially propagated in the network before a query started propagating.

The packet length of an agent or a query is assumed to remain the same as they propagate in the network. Since the energy spent on agents is fixed, to hold the total energy constant, the query energy also should be fixed. Given that the packet length is fixed, for example, if $N_a = 1$, the query uses 30 hops, then when $N_q = 2$ a query can go only 15 hops utilizing the same energy, and for $N_q = 3$ case for 10 hops, etc. As shown above $N_q = 1$ results in the highest $R(h_q)$, so let $N_q = 1$. From Fig. 12, $R(h_q)$ for $(N_a, N_q) = (1, 1)|_{N_a=30}$, $(N_a, N_q) = (1, 2)|_{N_q=15}$ and $(N_a, N_q) = (1, 3)|_{N_q=10}$ are the same, i.e. 0.4. This holds for all other $h_q$ values as well. Therefore, it can be concluded that for a given energy allocation for queries, $R(h_q)$ that can be achieved is independent of the number of queries, $N_q$.

Now let us investigate the effect of initially sent packet type on the resultant probability when there is an energy constraint. Performance of $(N_a, N_q) = (1, 3)$ is same as that of $(N_a, N_q) = (1, 3)$. But as observed in Fig. 13, the performance of $(N_a, N_q) = (1, 3)$ is not the same as that of $(N_a, N_q) = (3, 1)$. The variance of the number of nodes not receiving an agent is higher for $N_a = 3$, than that for $N_a = 1$, as each message propagates only one third of the distance in the former compared to the latter. First consider the case of sending out a single query. The probability of an agent not meeting a query is higher in $(N_a, N_q) = (3, 1)$ compared to that of $(N_a, N_q) = (1, 1)$. Therefore, the probability of a query meeting an agent is lower in $(N_a, N_q) = (3, 1)$ compared to that for $(N_a, N_q) = (1, 1)$. Moreover the probability of an agent not meeting query reduces as the number of queries increases, so that probability of a query meeting an agent for $(N_a, N_q) = (1, 3)$ is higher than that for $(N_a, N_q) = (1, 1)$. Hence, the probability of a query meeting an agent is lower in $(N_a, N_q) = (3, 1)$ compared to that for $(N_a, N_q) = (1, 3)$.

Next, for Scenario B, packet drop probability ($p_d$) is set to 0.2. All the other parameters and assumptions are same as in the previous analysis. Fig. 12 shows $R(h_q)$ as the number of queries and agents vary from 1 to 3 in a lossy network. As in the previous example, event agent(s) is sent out first and after agent(s) is spread completely the query(s) is sent. Fig. 13 shows the variation of $R(h_q)$ with cumulative hop count used by all the queries, which provides a fair and clear comparison as the number of agents varies, under fixed energy scenario. In contrast to the observations in the lossless network illustrated previously, in a lossy network with $p_d = 0.2$, the highest $R(h_q)$ is achieved when there are 2 agents and 1 query, as the number of queries and agents vary from 1 to 3. Table 4 gives the average reliability of a query meeting an agent anywhere in the network when the total number of hops used by queries is fixed at 30, and the number of queries is varied from 1 to 3.

### Table 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scenario A</th>
<th>Scenario B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_a$</td>
<td>170 for single agent</td>
<td>170/2 for each agent in double agent case</td>
</tr>
<tr>
<td>$H_b$</td>
<td>0–30 for constant energy compression performance at following hop counts were considered</td>
<td>30 for single agent</td>
</tr>
<tr>
<td>$N_a$</td>
<td>Varied from 1 to 3</td>
<td>Varied from 1 to 3</td>
</tr>
<tr>
<td>$N_q$</td>
<td>Varied from 1 to 3</td>
<td>Varied from 1 to 3</td>
</tr>
<tr>
<td>$p_d$</td>
<td>0</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Packet length Fixed

Energy of the agents $= E_a H_a$

where $E_a$ is the energy consumption per packet transmission in a single hop.

Without loss of generality, assume that the agent generation is equally likely to happen at any node. To evaluate the impact of $H$ on the rendezvous probability, we use simulation to evaluate Average reliability per node defined as,

$$R(h_q) = \frac{\text{Rendezvous probability of query and agent within } h_q \text{ hops}}{\text{Total number of nodes that agent can reach within TTL } H_a}$$

Fig. 11 illustrates the variation of $R(h_q)$ of a query meeting an agent within a given number of query hops, $h_q$. When total agent cost remains fixed, the performance degrades as the number of agents increases. This is due to the fact that for one agent (rumor), the variance of the probability of the agent visiting nodes in the network before a query is issued. Note that an agent and a query do not have to meet at a particular node at the same time as each of the nodes visited by an agent keeps a record about the event for future retrieval by a query.

The total energy consumption is the sum of the energy used for agents and the energy used for queries. First, we consider a network where the packet length is fixed, the packet loss is negligible, and the next node is equally likely to be any of the neighbors. In this analysis the total energy for the agents is fixed, i.e., if the number of agents is doubled, the TTL of each agent is halved. If the TTL is $H_a$ when $N_a = 1$, the energy consumption by agents is given by:

$$E_a H_a$$

Fig. 10. Probability of a packet visiting $(l, j)$ within $H$ hops with boundary conditions; analytical values using Eqs. (15) and (17), and Monte Carlo simulations.
Fig. 11. Variation of $R(h_q)$ with query hop count $(h_q)$ when both the number of queries and agents changes from 1 to 3 in a lossless network.

Table 4

<table>
<thead>
<tr>
<th>Case $(N_a, N_q)$</th>
<th>(1, 1)</th>
<th>(1, 2)</th>
<th>(1, 3)</th>
<th>(2, 1)</th>
<th>(2, 2)</th>
<th>(2, 3)</th>
<th>(3, 1)</th>
<th>(3, 2)</th>
<th>(3, 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(h_q)$</td>
<td>0.26</td>
<td>0.255</td>
<td>0.25</td>
<td>0.26</td>
<td>0.25</td>
<td>0.24</td>
<td>0.23</td>
<td>0.21</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Fig. 12. Variation of $R(h_q)$ with query hop count as the number of queries and agents changes from 1 to 3 in a lossy network.

Fig. 13. Variation of $R(h_q)$ with cumulative query hop count as number of queries and agents changes in a lossy network.

6. Conclusions

An exact mathematical model was presented for the rendezvous probability of agents and queries using random routing in networks based on grid topology. The equations derived facilitate the evaluation of many useful performance parameters for random routing, such as the probability of a query failing to find an informed node, the probability with which a node should generate the query to achieve a given service requirement, and the probability of a node locating an event without a priori event information. Thus the model can be used to select parameters for optimizing performance, and to evaluate the performance variation as these parameters vary. Examples presented demonstrated how to achieve the optimal point for fixed energy under both lossy and lossless network conditions. The model can be used similarly to evaluate the performance in terms of delay, reliability, etc. Finally, we have shown that the results from the model hold even for sparse networks with more than 75% node availability. The model thus can be applied to networks where the sleep/wake-up schedules of neighbors overlap significantly as well.

The main advantages of the model are its simplicity and adaptability. Further research is needed to handle $n$-connected networks and weighted random routing schemes where the probability of the next hop depends on other information such as network density, remaining node energy, previous agent/query information, etc. Extending the analysis to evaluate networks using geographical coordinates or logical coordinates to increase the performance through more constrained random routing is also of interest. Formal strategies need to be developed to handle finite networks with arbitrary boundary shapes.

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